FLUID FLOW BETWEEN A ROTATING DISK AND A FIXED WALL

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The results of a theoretical investigation of the fluid flow in the gap between a rotating disk and a fixed wall are presented. The variations of the rate of twisting of the fluid and the statistical pressure along the radius are analyzed for the case of centripetal flow.

A knowledge of the rate of twisting of the fluid flowing past the rotating disk and of its pressure change is necessary for computing the thermal state of turbine disks and also for a more accurate determination of the total temperature and the friction at the heat-exchange surface.

The flow between a rotating disk and a fixed wall in the presence of cross flow from the center to the periphery and from the periphery to the center has been investigated in a number of studies. For example, in [1] the particular case is investigated where the friction at the wall and the disk are equal and the azimuthal velocity of air at the midpoint of the gap is always smaller than that of the disk ($V_u/\omega r < 1$).

In real conditions the coefficients of friction at the wall and the disk cannot be equal. As shown in [2], the relative azimuthal velocity $V_u/\omega r$ can change in wide limits depending on the magnitude of the cross flow and, as is well known, the coefficients of friction can be equal only for $V_u/\omega r = 0.5$.

In [3] the problem is solved in a more general formulation, when the frictions at the wall and the disk are not equal. However, in this work the effect of compressibility of the fluid in the radial flow is not taken into consideration.

We consider the flow in a relatively large gap between a fixed wall and a rotating disk in the presence of cross flow both from the center to the periphery (centrifugal flow) and from the periphery to the center (centripetal flow) when the frictional drag from the radial component of the velocity can be disregarded. In view of the fact that an exact solution of this problem is not possible, here we carry out the investigation under the following simplifying assumptions:

the flow is taken to be one-dimensional without considering the secondary phenomena;

the coefficient of friction is constant along the radius;

the friction at the wall and the disk results only in a change of the momentum of the fluid in the azimuthal direction;

the compressibility of the fluid is taken into consideration only for the flow in the radial direction.

Under these assumptions the change in the moment of momentum of an annular element dr under the action of frictional forces at the wall and the disk can be expressed in the form:

A. For centripetal flow

$$m_s \frac{d}{dr}(rV_u) = \mp 2\pi r^2 \rho \xi_{\mathrm{d}} \frac{(\omega r - V_u)^2}{2} + 2\pi r^2 \rho \xi_{\mathrm{w}} \frac{V_u^2}{2}. \tag{1}$$

B. For centrifugal flow

$$m_s \frac{d}{dr}(rV_u) = \pm 2\pi r^2 \rho \xi_{\mathrm{d}} \frac{(\omega r - V_u)^2}{2} - 2\pi r^2 \rho \xi_{\mathrm{w}} \frac{V_u^2}{2}. \tag{2}$$

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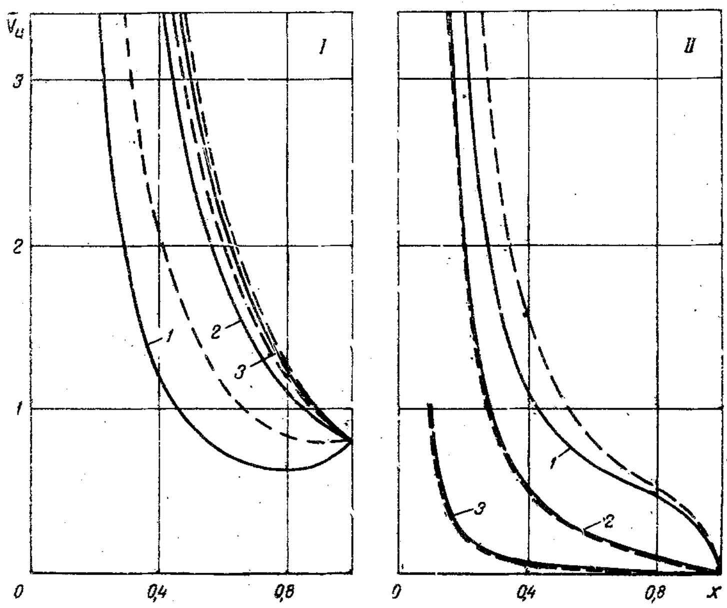

Fig. 1. Variation of relative twisting of the fluid along the radius in centripetal flow. Continuous lines, $\bar{\xi} = 1.5$; dashes, $\bar{\xi} = 0.667$: I) $\bar{V}_{u,1} = 0.8$; II) 0: 1) $a = 10$; 2) 1; 3) 0.1.

In these equations the sign of the first term on the right hand side is taken positive or negative depending on the magnitude and direction of the initial twisting of the fluid. Writing these equations in dimensionless form and introducing the notations

$$x = \frac{r}{r_1}, \quad \bar{\xi} = \frac{\xi_W}{\xi_d}, \quad a = \frac{2\pi \xi_d \omega r_1^2}{m_s}, \quad \bar{V}_u = \frac{V_u}{\omega r},$$

we obtain:

A. For centripetal flow

$$\bar{V}_{u,1} < 1, \ \bar{V}_{u,1} \geqslant 1$$

$$\mp \frac{d}{dx}(x^2 \bar{V}_u) = a\left[\frac{(x^2 - x^2\bar{V}_u)^2}{2} \mp \bar{\xi}\,\frac{x^4 \bar{V}_u^2}{2}\right]. \tag{3}$$

B. For centrifugal flow

$$\bar{V}_{u,0} \leqslant 1, \ \bar{V}_{u,0} > 1$$

$$\pm \frac{d}{dx}(x^2 \bar{V}_u) = a\left[\frac{(x^2 - x^2\bar{V}_u)^2}{2} \mp \bar{\xi}\,\frac{x^4 \bar{V}_u^2}{2}\right]. \tag{4}$$

Here

$$\bar{V}_{u,1} = \frac{V_{u,1}}{\omega r_1} \text{ and } \bar{V}_{u,0} = \frac{V_{u,0}}{\omega r_0}$$

is the initial relative twist of the fluid.

Equations (3) and (4) are Riccati type equations; as is well known, these equations cannot be solved directly in an analytical form.

Introducing the notation $y = x^2 \bar{V}_u$ for simplifying writing and using a substitution of the form

$$Z = \exp\left[\pm \int a\left(\frac{x^2 - y}{2} + \bar{\xi}\,\frac{y}{2}\right)dx\right], \tag{5}$$

after some manipulations we obtain the following differential equations:

$$\text{A. } \bar{V}_{u,1} < 1 \text{ and } \bar{V}_{u,1} \geqslant 1$$

$$Z'' \pm \left(ax - \bar{\xi}\,\frac{a^2}{4}x^4\right)Z = 0; \tag{6}$$

$$\text{B. } \bar{V}_{u,0} \leqslant 1 \text{ and } \bar{V}_{u,0} > 1$$

$$Z'' \mp \left(ax + \xi \frac{a^2}{4} x^4\right) Z = 0. \tag{7}$$

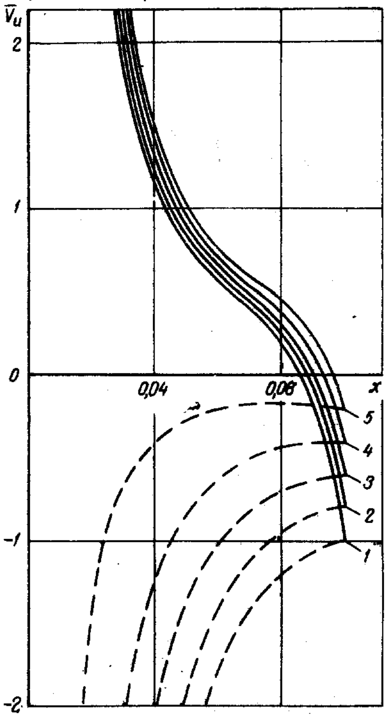

Fig. 2. Variation of relative twisting of the fluid along the radius in centripetal flow. Continuous curves a = 10; dashes, a = 1; $\bar{\xi}$ = 0.667: 1) $\bar{V}_{u,1}$ = −1; 2) −0.8; 3) −0.6; 4) −0.4; 5) −0.2.

As is well known, the general solution of a second order equation can be written in the form

$$Z(x) = C_1 Z_1(x) + C_2 Z_2(x). \tag{8}$$

Equations (6) and (7) are Whittaker type equations and can be solved using power series. Without discussing the characteristics of the method in detail, which can be found in any appropriate text book, we give only the final formulas for determining the rate of twisting at the midpoint of the gap:

$$\text{A. } \bar{V}_{u,1} < 1, \ \bar{V}_{u,1} \geqslant 1$$

$$\bar{V}_u = \frac{1}{(1 \mp \xi)} \left[1 \pm \frac{2}{ax^2} \frac{Z_1'(x) + DZ_2'(x)}{Z_1(x) + DZ_2(x)} \right]; \tag{9}$$

$$\text{B. } \bar{V}_{u,0} \leqslant 1, \ \bar{V}_{u,0} > 1$$

$$\bar{V}_u = \frac{1}{(1 \mp \xi)} \left[1 \mp \frac{2}{ax^2} \frac{Z_1'(x) + DZ_2'(x)}{Z_1(x) + DZ_2(x)} \right]. \tag{10}$$

Here after contraction by C_1 a single integration constant $D = C_2/C_1$ is obtained. Constant D must be determined from the following boundary conditions:

$x = 1$, $\bar{V}_u = \bar{V}_{u,1}$ for centripetal flow,

$x = x_0$, $\bar{V}_u = \bar{V}_{u,0}$ for centrifugal flow.

The above formulas can be used for computing the rate for all values of $\bar{V}_{u,1}$, $\bar{V}_{u,0}$, and $\bar{\xi}$ except the case $\bar{V}_{u,1} < 1$ and $\bar{\xi} = 1$ and also $\bar{V}_{u,0} \leq 1$ and $\bar{\xi} = 1$. For these values of $\bar{\xi}$ Eqs. (9) and (10) become inapplicable. These particular cases correspond to the conditions investigated in [1].

Computations using the results of [4] were carried out in order to check the formulas for $\bar{\xi} \neq 1$ and to control the correctness of the program compiled for the computer. In this connection it must be pointed out that in [4] and [5] fluid flow between two rotating disks with identical velocity was investigated. The same result can be obtained from the present investigation if we take $\bar{\xi} = 0$ (i.e., absence of friction at the fixed wall) and the effect of interaction with the rotating disk is doubled. From a formal point of view for these conditions Eqs. (6) and (7) reduce to the form investigated in [4] and [5].

A comparison of the results of the computation using the formulas of the present work with the results of [4] showed no disagreements.

For determining the pressure in the fluid flowing by the rotating disk and the fixed wall it is necessary to consider the change of the total momentum of the flow in the meridional plane.

In the cooling systems for the disks of modern gas turbines the amount of heating of the cooling air is relatively small and therefore in the first approximation the effect of the nonisothermal nature of the flow can be disregarded. Thus the momentum equation will have the form [4]

$$\frac{dP}{\rho} + V_r dV_r = \frac{V_u^2}{r} dr. \tag{11}$$

Using the continuity equation and the equation of state

$$\frac{d}{dr}(F \rho V_r) = 0, \tag{12}$$

$$\frac{dP}{P} = k \frac{d\rho}{\rho} \tag{13}$$

and assuming that the width of the gap remains constant along the radius, we obtain the following expression for computing the pressure distribution in the centripetal flow:

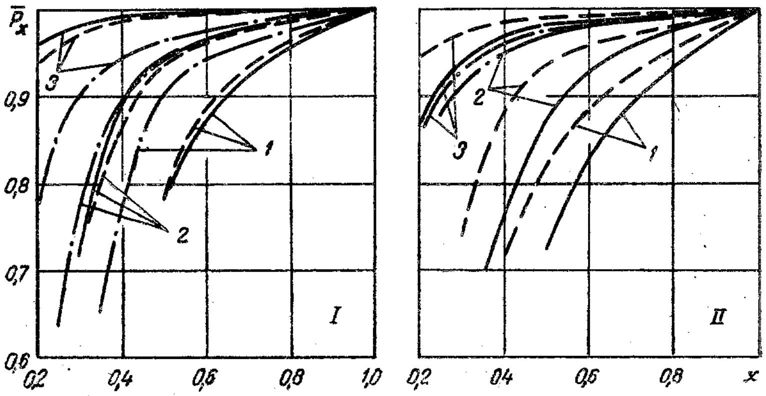

Fig. 3. Variation of relative pressure along the radius in centripetal flow. Continuous curves, $a = 0.1$; dashes, $a = 1.0$; dash-dots, $a = 10$; $\bar{\xi} = 0.667$: I) $1-\bar{V}_{u,1} = 0.8$; 2) 0.4; 3) 0; II) $1-\bar{V}_{u,1} = -1$; 2) -0.6; 3) -0.2.

$$M_{r,1}^2\left(\frac{1}{x^2\bar{P}_{x,1}^{2/k}}-1\right)+2M_{u,1}^2\int_x^1\bar{V}_u^2\,x\,dx=\frac{2}{k-1}\left(1-\bar{P}_{x,1}^{\frac{k-1}{k}}\right); \tag{14}$$

here the index 1 denotes the parameters corresponding to $r = r_1$ (or $x = 1$). Proceeding in the same way and denoting by index 0 the parameters corresponding to $r = r_0$, we obtain the computational formula for the centrifugal flow

$$M_{r,0}^2\left(1-\frac{x_0^2}{x^2}\bar{P}_{x,0}^{2/k}\right)+\frac{2M_{u,0}^2}{x_0^2}\int_{x_0}^{x}\bar{V}_u^2\,x\,dx=\frac{2}{k-1}\left(\frac{1}{\bar{P}_{x,0}^{\frac{k-1}{k}}}-1\right). \tag{15}$$

Here we have introduced the following dimensionless parameters:

$$\bar{P}_{x,1}=P_x/P_1,\quad \bar{P}_{x,0}=P_x/P_0,\quad M_{r,1}=V_{r,1}/a_{s0},$$
$$M_{u,1}=\omega r_1/a_{s0},\quad M_{r,0}=V_{r0}/a_{s0},\quad M_{u,0}=\omega r_0/a_{s0}.$$

In the case of flow of an incompressible fluid the corresponding formulas are

$$\bar{V}_{r,1}^2\left(\frac{1}{x^2}-1\right)+2\int_x^1\bar{V}_u^2\,x\,dx=\frac{2P_1}{\rho\omega^2 r_1^2}\left(1-\bar{P}_{x,1}\right) \tag{16}$$

for the centripetal flow and

$$\bar{V}_{r,0}^2\left(1-\frac{x_0^2}{x^2}\right)+\frac{2}{x_0^2}\int_{x_0}^{x}\bar{V}_u^2\,x\,dx=\frac{2P_0}{\rho\omega^2 r_0^2}\left(\frac{1}{\bar{P}_{x,0}}-1\right) \tag{17}$$

for the centrifugal.

Knowing the values of $\bar{P}_{x,1}$ and $\bar{P}_{x,0}$ the force acting on the rotating disk or the fixed wall can be determined.

In order to illustrate the method discussed above we carried out a numerical investigation for the case of centripetal flow. As is well known, this type of flow is encountered in the breaking of a twisted gas between turbine stages, in a sample of cooling air behind the blade of a centrifugal compressor, and in such other cases.

Three parameters a, $\bar{V}_{u,1}$, and $\bar{\xi}$ affect the nature of distribution of the relative velocity along the radius of the disk.

The graph of the variation of the relative rate of twisting of the fluid in the gap is shown in Fig. 1I for three values of parameter a, differing by an order of magnitude. It is evident from the graph that for $a \leq 1$ for large mass flow rates with other conditions remaining equal the nature of variation of the relative rate along the radius remains practically unchanged. This result is explained by the fact that the initial moment of momentum of the fluid flowing into the gap is sufficiently large compared to the work done by frictional forces at the wall and disk. Therefore the nature of variation of the relative rate of twisting is close to the law $\bar{V}_u x^2$ = const.

With the decrease in the flow rate ($a > 1$) the work of the frictional forces may cause significant change in the nature of twisting of the fluid along the radius; with the increase of the drag coefficient at the fixed wall compared to the rotating disk ($\bar{\xi} > 1$) a noticeable decrease of the rate of twisting is observed in the peripheral part.

On decreasing the initial rate of twisting to zero the fluid gets twisted due to the frictional forces at the disk. The distribution of the relative twisting along the radius is shown in Fig. 1II for this case. It is evident from the graph that the relative twisting of the fluid increases with the decrease of the flow rate of air; at large flow rates of air ($a \leq 1$) the main drag comes from overcoming the forces of inertia of the air itself and the drag forces at the fixed wall have practically insignificant role.

The fluid flowing into the gap may have initial twisting opposite to the rotation of the disk. The results of computations for this case are given in Fig. 2. As can be seen, at large flow rates ($a = 1$) the fluid rotates in the direction opposite to the rotation of the disk; with the increase of the initial twist the disk begins to exert a noticeable retarding effect. With the decrease of the flow rate ($a = 10$) the retarding action of the disk increases so much that the fluid changes its initial direction and begins to rotate with the disk.

The change in the relative static pressure was also computed from the computed values of the relative rate of twisting of the fluid in the gap (Fig. 3I, II).

It is evident that an increase of the initial twisting and the flow rate of the fluid through the gap between the rotating disk results in a sharper change of the static pressure along the radius and vice versa.

Similar computations were done also for centrifugal flow, which showed that with the inflow of a non-twisted fluid a small change of the static pressure occurs, which is caused by a relatively small twisting of the fluid from the rotating disk.

Thus the present investigation shows that a sharp decrease of the static pressure may occur in the centripetal flow of air or gas between a rotating disk and a fixed wall. The use of this phenomenon would permit to increase the efficiency of laboratory condensation between turbine stages by blowing in cold air into the peripheral region with twist in the direction of rotation of the disk.

NOTATION

x	is the dimensionless radius;
a	is the dimensionless parameter;
r_1	is the maximum radius of the disk along which the flow occurs;
r_0	is the initial radius of supply of the fluid between the rotating disk and the fixed wall;
r	is the instantaneous value of the radius;
V_u	is the rate of twisting of the fluid at the midpoint of the gap;
$\bar{V}_u$	is the relative rate of twisting of the fluid in the gap at radius x;
$\bar{V}_{u,1}$	is the initial circular velocity of the fluid in centripetal flow;
$\bar{V}_{u,0}$	is the initial circular velocity in centrifugal flow;
P_1, P_0	are the initial pressure in centripetal and centrifugal flow, respectively;
P	is the static pressure of the medium at a radius r;
$P_{x,1}$, $P_{x,2}$	are the relative static pressures of the medium for centripetal and centrifugal flow, respectively;
ρ	is the density of the fluid;
ξ_d	is the coefficient of friction at the disk;
ξ_w	is the coefficient of friction at the wall;
$\bar{\xi}$	is the relative coefficient of friction at the wall;
ω	is the angular speed of rotation of the disk;
m_s	is the mass flow rate of the fluid through the cavity;
C_1, C_2	are the integration constants.

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